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ON THE FREE VIBRATION ANALYSIS OF A SANDWICH BEAM WITH TIP MASS

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ABSTRACT

This paper presents the free vibration analysis of a sandwich beam with a tip mass using higher order sandwich panel theory (HSAPT). The governing equations of motion and boundary conditions are obtained using Hamilton's principle. General Differential Quadrature (GDQ) is employed to solve the system governing equations of motion. The natural frequencies and mode shapes of the system are presented and Ansys simulation is performed to validate the results. Various boundary conditions are also employed to examine the natural frequencies of the sandwich beam without tip mass and the results are compared with those found in the literature. Parametric studies are conducted to examine the effect of key design parameters on the natural frequencies of the sandwich beam with and without tip mass.

INTRODUCTION

Sandwich structures find applications in many fields of engineering because of their interesting properties such as high stiffness and strength, low weight, and high energy absorption capability. Their use is increasingly observed in the aerospace, civil constructions, naval, and automotive industries [1]. Sandwich structures usually consist of two outer layers and one middle layer. The middle layer or core is bounded by the outer layers also known as face sheets. The core can be made of honeycomb, balsa wood, spatial metallic foam, polymeric foam, or other lightweight materials. The construction of sandwich structures can be classified into two categories depending on the relative stiffness of the core with respect to the face sheets. When high stiffness-to-weight ratio is desired, the core is made stiffer and thicker than the face sheets. When vibration reduction is the primary goal, the core should be softer and thinner than the face sheets [2].

Many theoretical models of sandwich structures have been proposed in the last few decades based on the type of core, which in general dictates the physical behavior of the sandwich structure. The obtained models can be broadly grouped into two categories: classical [2-3] and modern [4-7]. The former was established based on Euler-Bernoulli theory and assuming infinite core stiffness. Some classical models employed Timoshenko beam theory by considering the transverse shear deformation of the core, but assuming linear longitudinal deformation in the thickness and infinite stiffness in the transverse direction. Such models are known as First Order Shear Deformation Theory (FOSDT) and are usually acceptable when the sandwich structure is statically loaded and vertical very stiff [4]. However, modern sandwich panels with cores of foam type are flexible in all directions and thus violating the stiff core assumption of classical models.

Modern approaches use higher order displacement models known as high-order sandwich panel theory (HSAPT) [4-7]. Frostig and Baruch represented the HSAPT mixed formulation in term of displacements in various skins, longitudinal and vertical, and shear stresses in the core [4]. The other type of HSAPT is a displacement-based formulation in which the aforementioned four displacement variables of the HSAPT (mixed) are the same, but the core shear stress variable is replaced by the mid-core transverse displacement, and the acceleration in the core is allowed to be nonlinear through the core [7].

Various numerical methods can be used for tackling static or dynamic problems of sandwich structures such as the Dynamic

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stiffness method (DSM) [8], classical Finite element method (FEM) [9-12] and General Differential Quadrature method [13-15]. Among them the GDQ method is much simpler, accurate and faster [16].

The GDQ method and HSAPT (displacement) theory are adopted in the present study to examine the free vibration of a sandwich beam with tip mass. It should be noted that the majority of the papers reported in the literature focus on bare sandwich beams [17], or on a regular beam with tip mass [18-19]. There is no work in the literature that examines the vibration of a sandwich beam with tip mass. The investigation of this problem is reported in this paper for the first time. The governing equations of motion are obtained using Hamilton's principle and free vibration analysis is conducted using the GDQ method [20]. The results of the GDQ method are validated using Ansys simulation. Parametric studies are carried out to investigate the effect of key design variables on the natural frequency of the system with and without tip mass.

MATHEMATICAL MODELING

A schematic diagram of the sandwich beam with tip mass is shown in Fig.1. The length and the width of the beam are, L and b respectively. This beam is composed of three layers. Two thin/stiff face sheets with thickness h_f and a thick/soft core with a thickness h_c. The length of the tip mass is defined as L_M. Here x and y denote the coordinate system directions.

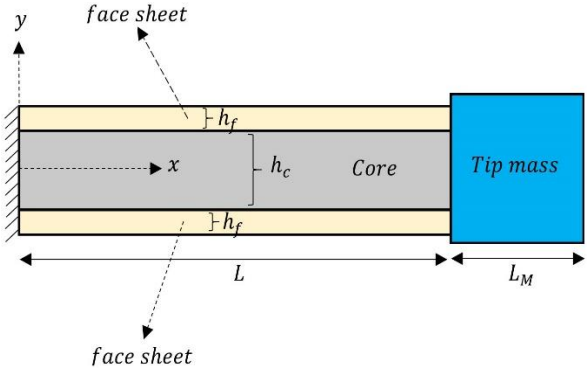

Figure 1. Schematic of sandwich beam with tip mass.

The kinetic (T) and potential (U) energies of the system depicted in Fig.1 are given as:

$$T = T_{beam} + T_{tip\ mass} = \frac{1}{2}\int \rho_f[(\dot{u}_t - y\dot{w}_t')^2 + \dot{w}_t^2]\,dV_t + \frac{1}{2}\int \rho_f[(\dot{u}_b - y\dot{w}_b')^2 + \dot{w}_b^2]dV_b + \frac{1}{2}\int \rho_c\left[\left(\dot{u}_c + \dot{\phi}_1 y + \left(\dot{u}_t + \dot{u}_b + \frac{h_f}{2}\dot{w}_t' - \frac{h_f}{2}\dot{w}_b' - 2\dot{u}_c\right)\frac{2y^2}{h_c^2} + \left(\dot{u}_t - \dot{u}_b + \frac{h_f}{2}\dot{w}_t' + \frac{h_f}{2}\dot{w}_b' - h_c\dot{\phi}_1\right)\frac{4y^3}{h_c^3}\right)^2 + \left(\dot{w}_c + \frac{(\dot{w}_t+\dot{w}_b)y}{h_c} + \frac{(\dot{w}_t+\dot{w}_b-2\dot{w}_c)2y^2}{h_c^2}\right)^2\right]dV_c + \frac{1}{2}M\left(\dot{w}_c(L,t) + \frac{L_M}{2}\dot{w}_c'(L,t) + \right)^2 + \frac{1}{2}M\dot{u}_c^2(L,t) + \frac{1}{2}J_0\dot{w}_c'(L,t)^2 \quad (1)$$

$$U = \frac{1}{2}\int_0^L\{\int E_f\,(u_t' - yw_t')^2 dA_t + \int E_f(u_b' - yw_b')^2 dA_b + \int E_c\left(\frac{w_t - w_b}{h_c} + \frac{(w_t+w_b-2w_c)4y}{h_c^2}\right)^2 dA_c + \int G_c[(\phi_1 - w_c') + (u_t + u_b + \frac{2h_f+h_c}{4}w_t' - \frac{2h_f+h_c}{4}w_b' - 2u_c)\frac{4y}{h_c^2}dA_c + (u_t - u_b + \frac{3h_f+h_c}{6}w_t' + \frac{3h_f+h_c}{6}w_b' - h_c\phi_1 - \frac{h_c}{3}w_c')\frac{12y^2}{h_c^3}]^2\,dA_c\}dx \quad (2)$$

Where ρ_f and ρ_c are the density of the face sheets and core, M is the mass of tip mass. The terms u_c , u_t , u_b are longitudinal displacements at core, top and bottom face sheets and w_c , w_t , w_b are vertical displacements at core, top and bottom face sheets. Here ϕ_1 is also the slope at the centroid of the core, w_t' and w_b' are the displacements of top and bottom face sheets. Here, E_f, and E_c denotes the modulus of elasticity of face sheets and the core, respectively, and $G_c$is the shear modulus of the core. A_t, A_b, and A_c are the areas of top, bottom, and core in x direction. V_t, V_b, and V_c are also the volumes of top, bottom, and core layers. The governing equations of motion are obtained using Hamilton's principle and are given by the following seven equations (Eqs.3-9):

$$m_f\ddot{u}_t + m_c[\frac{1}{20}(\ddot{u}_t + \ddot{u}_b) + \frac{1}{28}(\ddot{u}_t + \ddot{u}_b) + \frac{1}{15}\ddot{u}_c + \frac{1}{70}h_c\ddot{\phi}_1 + \frac{1}{56}h_f(\ddot{w}_t' - \ddot{w}_b') - E_fA_fu_t'' + \frac{G_cA_c}{h_c^2}[\frac{4}{3}(u_t + u_b) + \frac{9}{5}(u_t - u_b) - \frac{8}{3}u_c - \frac{4}{5}h_c\phi_1 + \left(\frac{2}{3}h_f + \frac{1}{3}h_c\right)(w_t' - w_b') + \left(\frac{9}{10}h_f + \frac{3}{10}h_c\right)(w_t' + w_b') - \frac{2}{5}h_cw_c'] = 0 \quad (3)$$

$$m_f\ddot{u}_b + m_c[\frac{1}{20}(\ddot{u}_t + \ddot{u}_b) - \frac{1}{28}(\ddot{u}_t + \ddot{u}_b) + \frac{1}{15}\ddot{u}_c - \frac{1}{70}h_c\ddot{\phi}_1 + \frac{1}{40}h_f(\ddot{w}_t' - \ddot{w}_b') - \frac{1}{56}h_f(\ddot{w}_t' + \ddot{w}_b') - E_fA_fu_b'' + \frac{G_cA_c}{h_c^2}\left(\frac{4}{3}(u_t + u_b) - \frac{9}{5}(u_t - u_b) - \frac{8}{3}u_c + \frac{4}{5}h_c\phi + \left(\frac{2}{3}h_f + \frac{1}{3}h_c\right)(w_t' - w_b') - \left(\frac{9}{10}h_f + \frac{3}{10}h_c\right)(w_t' + w_b') - \frac{2}{5}h_cw_c'\right] = 0 \quad (4)$$

$$m_c\left[\frac{8}{15}\ddot{w}_c + \frac{1}{15}(\ddot{w}_t + \ddot{w}_b)\right] + \frac{G_cA_c}{h_c^2}\left[-\frac{2}{5}h_c(u_t' - u_b') - \frac{4}{15}h_c^2\phi_1 - \left(\frac{1}{5}h_fh_c + \frac{1}{5}h_c^2\right)(w_t'' + w_b'') - \frac{8}{15}h_c^2w_c''\right] + \frac{E_cA_c}{h_c^2}[-\frac{8}{3}(w_t + w_b) + \frac{16}{3}w_c] = 0 \quad (5)$$

$$m_c(\frac{8}{15}\ddot{u}_c + \frac{1}{15}h_f^2(\ddot{u}_b + \ddot{u}_t) + \frac{1}{30}h_f\left(\ddot{w}_t' - \ddot{w}_b'\right) + \frac{G_cA_c}{h_c^2}\left[-\frac{8}{3}(u_t + u_b) + \frac{16}{3}u_c - \left(\frac{4}{3}h_f + \frac{2}{3}h_c\right)(w_t' - w_b')\right] = 0 \quad (6)$$

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$$m_c[\frac{2}{105}h_c^2\ddot{\phi}_1+\frac{1}{70}h_c\,(\ddot{u}_t+\ddot{u}_b)+\frac{1}{30}h_fh_c(\ddot{w}_t'-\ddot{w}_b')]+\frac{G_cA_c}{h_c^2}\left[-\frac{8}{3}(u_t-u_b)+\frac{16}{3}u_c-\left(\frac{4}{3}h_f+\frac{2}{3}h_c\right)(w_t'-w_b')\right]=0 \tag{7}$$

$$m_f\left(\ddot{w}_t-\frac{1}{2}h_f^2\ddot{w}_t''\right)+m_c[-\frac{1}{40}h_f(\ddot{u}_t'+\ddot{u}_b')-\frac{1}{56}h_f(\ddot{u}_t'-\ddot{u}_b')-\frac{1}{30}h_f\ddot{u}_c'-\frac{1}{140}h_fh_c\ddot{\phi}_1'-\frac{1}{80}h_f^2(\ddot{w}_t''-\ddot{w}_b'')+\frac{1}{12}(\ddot{w}_t-\ddot{w}_b)-\frac{1}{112}h_f^2(\ddot{w}_t''+\ddot{w}_b'')+\frac{1}{20}(\ddot{w}_t+\ddot{w}_b)+\frac{1}{15}\ddot{w}_c]+E_fI_fw_t''''+\frac{G_cA_c}{h_c^2}[-\left(\frac{2}{3}h_f+\frac{1}{3}h_c\right)(u_t'+u_b')-\left(\frac{9}{10}h_f+\frac{3}{10}h_c\right)(u_t'-u_b')+\left(\frac{4}{3}h_f+\frac{2}{3}h_c\right)u_c'+\left(\frac{2}{5}h_fh_c+\frac{2}{15}h_c^2\right)\phi_1'-\left(\frac{9}{20}h_f^2+\frac{3}{10}h_fh_c+\frac{1}{20}h_c^2\right)(w_t''+w_b'')-\left(\frac{1}{3}h_fh_c+\frac{1}{3}h_f^2+\frac{1}{12}h_c^2\right)(w_t''-w_b'')-\left(\frac{1}{5}h_fh_c+\frac{1}{15}h_c^2\right)w_c'']+\frac{E_cA_c}{h_c^2}\left[\frac{4}{3}(w_t+w_b)+(w_t-w_b)-\frac{8}{3}w_c\right]=0 \tag{8}$$

$$m_f\left(\ddot{w}_b-\frac{1}{2}h_f^2\ddot{w}_b''\right)+m_c[-\frac{1}{40}h_f(\ddot{u}_t'+\ddot{u}_b')-\frac{1}{56}h_f(\ddot{u}_t'+\ddot{u}_b')+\frac{1}{30}h_f\ddot{u}_c'-\frac{1}{140}h_fh_c\ddot{\phi}_1'+\frac{1}{80}h_f^2(\ddot{w}_t''+\ddot{w}_b'')-\frac{1}{12}(\ddot{w}_t-\ddot{w}_b)-\frac{1}{112}h_f^2(\ddot{w}_t''+\ddot{w}_b'')+\frac{1}{20}(\ddot{w}_t+\ddot{w}_b)+\frac{1}{15}\ddot{w}_c]+E_fI_fw_b''''+\frac{G_cA_c}{h_c^2}[\left(\frac{2}{3}h_f+\frac{1}{3}h_c\right)(u_t'+u_b')-\left(\frac{9}{10}h_f+\frac{3}{10}h_c\right)(u_t'-u_b')-\left(\frac{4}{3}h_f+\frac{2}{3}h_c\right)u_c'+\left(\frac{2}{5}h_fh_c+\frac{2}{15}h_c^2\right)\phi_1'-\left(\frac{9}{20}h_f^2+\frac{3}{10}h_fh_c+\frac{1}{20}h_c^2\right)(w_t''+w_b'')+\left(\frac{1}{3}h_fh_c+\frac{1}{3}h_f^2+\frac{1}{12}h_c^2\right)(w_t''-w_b'')-\left(\frac{1}{5}h_fh_c+\frac{1}{15}h_c^2\right)w_c'']+\frac{E_cA_c}{h_c^2}[\frac{4}{3}(w_t+w_b)-(w_t-w_b)-\frac{8}{3}w_c] \tag{9}$$

The boundary conditions are given as

$$X=0\begin{cases}u_t(0,t)\,,P_t(L,t)=0\\ w_t(0,t)\,,V_t(L,t)=0\\ w_t'(0,t)\,,M_t(L,t)=0\\ w_c(0,t)\,,V_c(L,t)=0\\ u_b(0,t)\,,P_b(L,t)=0\\ w_b(0,t)\,,V_b(L,t)=0\\ w_b'(0,t)\,,M_b(L,t)=0\end{cases} \tag{10}$$

The general forms of the axial forces (P_t and P_b), the shear forces (V_t, V_b, V_c) and bending moments (M_t and M_b) are given as

$$P_t=-E_fA_f\frac{\partial u_t}{\partial x}-\left[\frac{M}{4}+\frac{ML_M^2}{4(h_f+h_c)^2}+\frac{J_0}{(h_f+h_c)^2}\right]\ddot{u}_t+\left[-\frac{M}{4}+\frac{ML_M^2}{4(h_f+h_c)^2}+\frac{J_0}{(h_f+h_c)^2}\right]\ddot{u}_b+\frac{ML_M}{2(h_f+h_c)}\ddot{w}_c \tag{11}$$

$$P_b=-E_fA_f\frac{\partial u_b}{\partial x}+\left[-\frac{M}{4}+\frac{ML_M^2}{4(h_f+h_c)^2}+\frac{J_0}{(h_f+h_c)^2}\right]\ddot{u}_t-\left[\frac{M}{4}+\frac{Ml_M^2}{4(h_f+h_c)^2}+\frac{J_0}{(h_f+h_c)^2}\right]\ddot{u}_b-\frac{ML_M}{2(h_f+h_c)}\ddot{w}_c \tag{12}$$

$$V_t=-\frac{1}{12}m_fh_f^2\ddot{w}_t'+\frac{m_c}{840}(-36\,h_f\ddot{u}_t-6h_f\ddot{u}_b-28h_f\ddot{u}_c-6h_fh_c\ddot{\phi}_1+3\,h_f^2\ddot{w}_b'-18\,h_f^2\ddot{w}_t')+E_fI_fw_t''''+\frac{G_cA_c}{840}h_c^2[(196\,h_f-28h_c)u_b-(1316h_f+532\,h_c)u_t+(560h_c+1120h_f)u_c+(28h_c^2-98\,h_f^2+28\,h_fh_c)w_b''-(112h_c^2+658\,h_f^2+532\,h_f^2+532\,h_fh_c)w_t'+(336\,h_fh_c+112h_c^2)\phi_1-(56h_c^2+168\,h_fh_c)w_c'] \tag{13}$$

$$V_b=-\frac{1}{12}m_fh_f^2\ddot{w}_b'+\frac{m_c}{840}(36\,h_f\ddot{u}_b+6h_f\ddot{u}_t+28h_f\ddot{u}_c-6h_fh_c\ddot{\phi}_1+3\,h_f^2\ddot{w}_t'-18\,h_f^2\ddot{w}_b')+E_fI_fw_b''''+\frac{G_cA_c}{840}h_c^2[(-196\,h_f+28h_c)u_t+(1316h_f+532\,h_c)u_b-(560h_c+1120h_f)u_c+(28h_c^2-98\,h_f^2+28\,h_fh_c)w_t''-(112h_c^2+658\,h_f^2+532\,h_f^2+532\,h_fh_c)w_b'+(336\,h_fh_c+112h_c^2)\phi_1-(56h_c^2+168\,h_fh_c)w_c'] \tag{14}$$

$$V_c=\frac{G_cA_c}{840\,h_c^2}[-336h_f(u_t-u_b)-224h_c^2\phi_1-(56h_c^2+168\,h_fh_c)(w_t'-w_b')-448h_c^2w_c']-M\ddot{w}_c-\frac{ML_M}{2}\frac{\ddot{u}_b-\ddot{u}_t}{h_f+h_c} \tag{15}$$

$$M_t=-E_fI_fw_t'' \tag{16}$$

$$M_b=-E_fI_fw_b'' \tag{17}$$

NUMERICAL RESULTS AND DISCUSION

The numerical simulation is based on the material properties listed in Tables 1 and 2 for the sandwich beam with and without tip mass, respectively. The first set of the numerical analyses is to validate the present formulation by comparing the obtained natural frequencies of the sandwich beam with tip mass using the GDQ method to those of Ansys simulation. The results are presented in Tables 3 and show very good agreement. To further validate the performance of the proposed formulation, Table 4

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compares the natural frequencies of a bare sandwich beam of the GDQ method to those in [21] considering various boundary conditions. The results also show very good agreement.

Table 1: Material properties of the sandwich beam with tip mass

E_f (Gpa)	G_c (Mpa)	ρ_f ($\frac{kg}{m^3}$)	ρ_c ($\frac{kg}{m^3}$)	h_f (mm)
210	22.1	7900	60	3
h_c (mm)	L (mm)	ρ_M ($\frac{kg}{m^3}$)	L_M (mm)	
40	260	2600	60	

Table 2: Material properties of the sandwich beam without tip mass for different boundary conditions

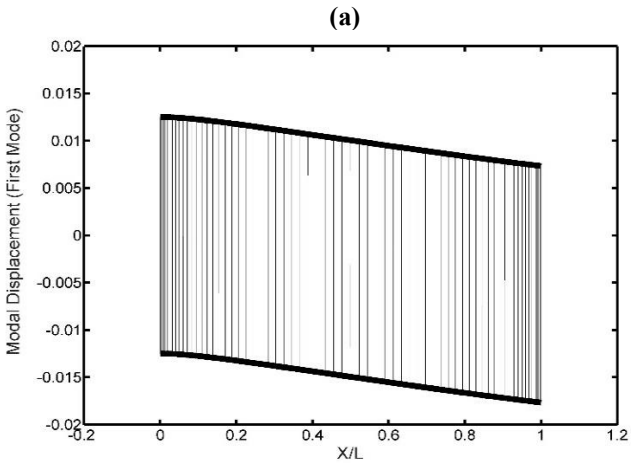
E_f (Gpa)	G_c (Mpa)	ρ_f ($\frac{kg}{m^3}$)	ρ_c ($\frac{kg}{m^3}$)	h_f (mm)
13.6	12.8	1800	58.5	5
h_c (mm)	L (mm)	ρ_M ($\frac{kg}{m^3}$)	L_M (mm)	
19	152.4	0	0	

Table 3: Natural frequencies of sandwich beam with tip mass (Hz)

Data	1	2	3	4	5
GDQ	89.18	354.73	731.86	1142.57	1715.59
Ansys	89.09	352.23	739.07	1195.20	1724.10

Table 4: Natural frequencies of sandwich beam without tip mass for different boundary conditions

Data		1	2	3	4	5
C-F	Present	329.18	1083.22	2152.54	2201.58	2258.15
	[21]	327.40	1079.10	2138.00	2175.30	2216.40
C-C	Present	844.17	1038.22	2152.54	2201.58	2258.15
	[21]	832.92	1079.10	2138.00	2175.30	2216.40
S-S	Present	619.05	1384.25	1508.81	2212.64	2413.09
	[21]	618.56	1349.10	1503.30	2170.40	2358.40
S-C	Present	717.25	1725.64	2233.90	2541.30	3157.28
	[21]	715.45	1714.40	2196.40	2514.40	3117.10



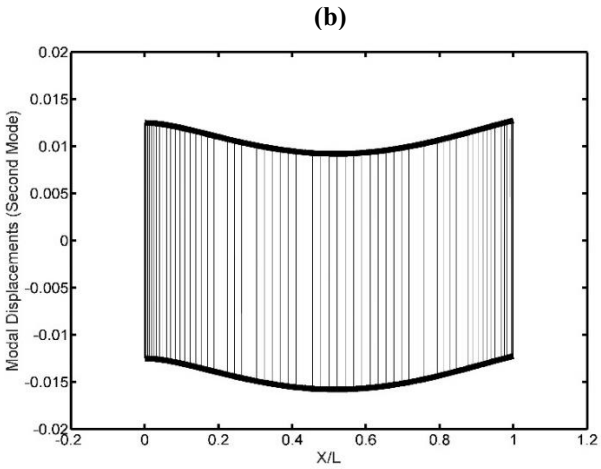


Figure 2: Mode shapes of sandwich beam with tip mass **(a)** first mode, **(b)** second mode.

Figure 2 depicts the first two mode shapes of sandwich beam with tip mass. Figure 3-5 also present the first two mode shapes of clamped-clamped, cantilever, and simply-simply supported boundary conditions, respectively. From Figures 2 and 3 it can be seen that the second mode shape of the sandwich beam with tip mass resembles to the first mode shape of Clamped-Clamped sandwich beam and the reason might be related to the effect of tip mass on right-hand side of sandwich beam. Figure 4 depicts the first two mode shapes of a cantilever sandwich beam without tip mass. Figure 5 presents the first two mode shapes of sandwich beam with simply supported edges.

Tables 5 - 7 show the effects of some of the key design parameters on the natural frequencies of a sandwich beam with and without tip mass. As expected, Table 5 shows that natural frequency increases with increasing core thickness for both sandwich beam with and without tip mass. However, the percentage of increase is more pronounced for the beam with tip mass. The role of the face sheet thickness on the natural frequencies is demonstrated in Table 6. For both sandwich beam with and without tip mass, the results indicate that natural frequency can increase or decrease with increasing face sheet thickness. Part of the reason for this can be attributed to the fact that the face sheets thickness has similar effect on both mass and stiffness; whereas, the core thickness has more effect on the stiffness of the beam. Table 7 demonstrates the role of the sandwich beam width on the natural frequencies. It can be observed that the width has no effect on the natural frequencies of the bare sandwich beam (i.e. no tip mass), but it has a significant effect on the natural frequencies of the loaded sandwich beam (i.e., beam with tip mass). This can be also seen from the boundary conditions of the sandwich beam with tip mass in which the beam width (*b*) clearly appears unlike for the sandwich beam without tip mass boundary condition in which the width vanishes after algebraic simplification. Overall, the results of the parametric studies indicate that the key design

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parameters have more effect on the dynamic of the loaded sandwich beam than that of the bare sandwich beam.

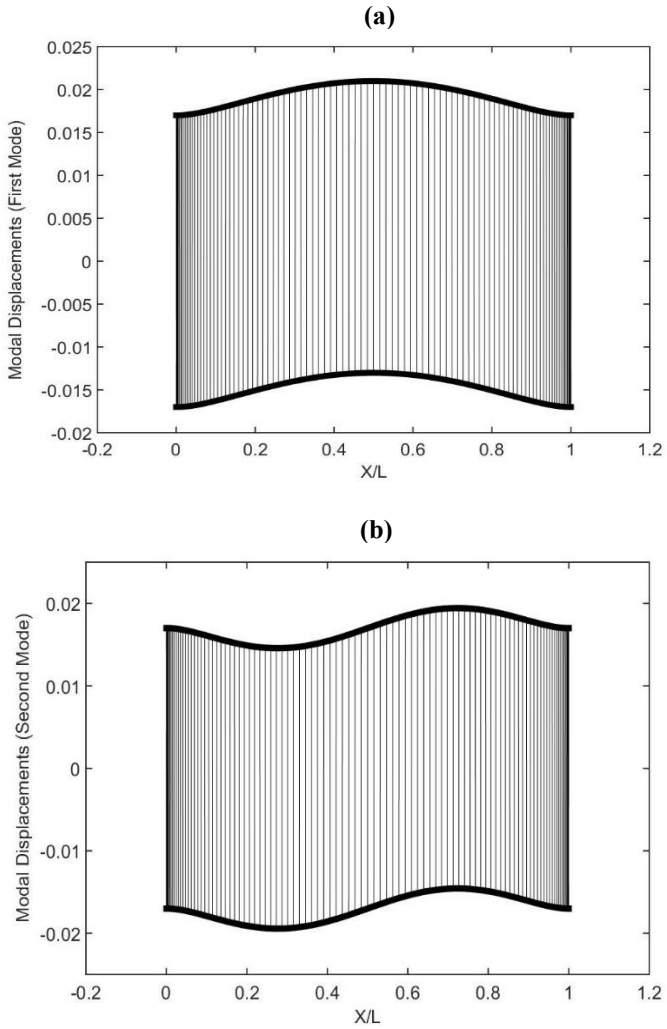
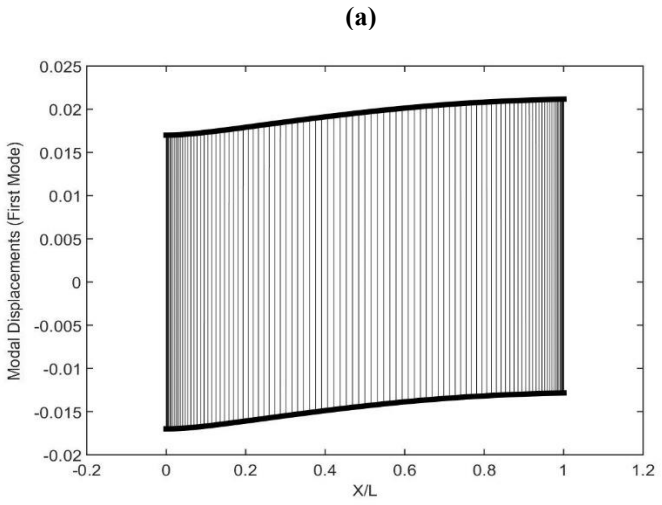


Figure 3. Mode shapes of clamped-clamped sandwich beam without tip mass **(a)** first mode, **(b)** second mode.



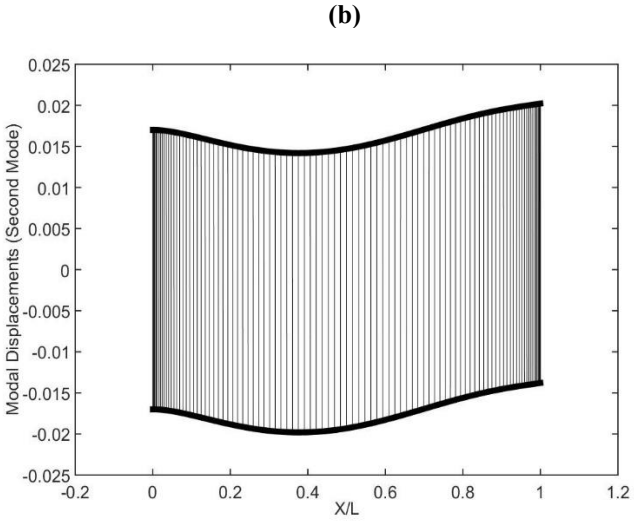


Figure 4. Mode shapes of cantilever sandwich beam without tip mass **(a)** first mode, **(b)** second mode.

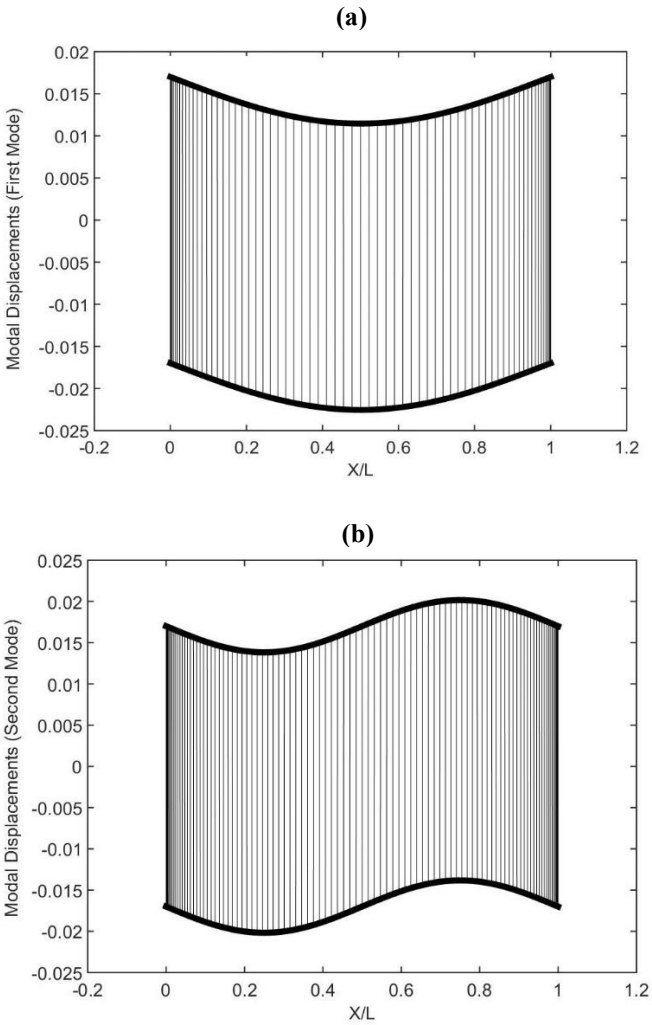


Figure 5. Mode shapes of Simply-Simply supported sandwich beam **(a)** first mode, **(b)** second mode.

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Table 5: The lowest natural frequencies (Hz) for sandwich beam with and without tip mass for different core thicknesses.

Modes	With tip mass		Without tip mass	
	$h_c = 10$	$h_c = 15$	$h_c = 10$	$h_c = 15$
1	53.81	61.48	100.98	112.68
2	250.95	272.49	363.85	392.52
3	598.27	628.41	792.41	825.18
4	953.37	1046.68	1410.43	1442.66
5	1616.23	1652.17	2232.67	2262.13

Table 6: The lowest natural frequencies (Hz) for sandwich beam with and without tip mass for different face sheet thicknesses.

Modes	With tip mass		Without tip mass	
	$h_f = 1.5$	$h_f = 3.5$	$h_f = 1.5$	$h_f = 3.5$
1	86.52	90.37	186.82	151.00
2	393.97	357.28	568.22	502.84
3	776.37	744.41	978.99	1005.22
4	1173.36	1157.08	1411.38	1610.83
5	1587.19	1590.45	1886.65	1714.15

Table 7: The lowest natural frequencies (Hz) for sandwich beam with and without tip mass for different width of the beam.

Modes	With tip mass		Without tip mass	
	$b = 40$	$b = 70$	$b = 40$	$b = 70$
1	76.72	93.99	154.82	154.82
2	343.66	359.86	501.74	501.74
3	722.40	736.52	959.083	959.083
4	1135.18	1146.31	1574.265	1574.265
5	1647.43	1650.42	1715.59	1715.59

CONCLUSION

In this study, we presented for the first time the free vibration of cantilever sandwich beam with tip mass and without tip mass for different boundary conditions using Higher Order Sandwich Panel Theory. The equations of motion and boundary conditions are obtained using Hamilton's principle. The partial differential equations were transformed into ordinary differential equations using the GDQ method.

Numerical analyses were conducted to determine the natural frequencies and mode shapes of the sandwich beam with and without tip mass. The results of the GDQ method were validated using Ansys simulation and showed very good agreement. Various boundary conditions were also used to examine the natural frequencies of the sandwich beam without tip mass and the results were compared to those found in the literature. Parametric studies were conducted to analyze the effect of key geometric parameters on the natural frequencies of the systems. It was demonstrated that the studied geometric parameters (i.e., width and thickness of core and face sheets) have more effect on the dynamic of the sandwich beam with tip mass that that of bare sandwich beam. The authors anticipate future work to be focused on experiments and forced vibration analyses of sandwich beam with tip mass.

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