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ON THE NONLINEAR VIBRATION ANALYSIS OF ULTRA PRECISION MANUFACTURING MACHINES WITH MODE COUPLING

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ABSTRACT

Nonlinear free undamped vibrations are investigated for ultra-precision manufacturing (UPM) machines with quadratic stiffness. The modes of the system are linearly coupled. The non-resonant case and the bounded internal resonance case are considered. The results of the non-resonant case indicate that the behavior of the system is the same as the linear behavior. However, for the internal resonance case, the results show that the amplitudes are coupled. The results also indicate that the nonlinear frequencies and amplitudes depend not only on the initial conditions, but also on the location of the isolators with respect to the center of gravity of the UPM.

INTRODUCTION

Micro and Nano products have received considerable attention by numerous researchers in the last few years. These products can be fabricated by Ultra-precision manufacturing (UPM) machines [1]. Since the tolerance and accuracy of these products are of high importance, the machines must be properly isolated from exogenous vibrations [2], [3].

Passive isolators are often used in order to reduce the vibration transmitted to the UPM machines because of their cost effectiveness and reliability [2], [3]. Decoupled vibration modes of the isolated machine can be achieved by aligning the isolator mounting location with the center of gravity (C.G) [4]. This decoupling prevents transmitting the vertical vibration to the horizontal axes of machine [3], [5]. It also eliminates the occurrence of two or more resonance peaks in the transmissibility response of machine [6]. However, aligning the C.G exactly with isolators is very difficult, so vibration modes are coupled in many UPM machine designs [3]. Furthermore, reduction of residual vibrations which resulted from soft isolator mounts can be achieved by coupling the vibration modes [7].

Udwadia [8] postulated that mode coupling changes the natural frequencies of the system and damping behavior. Chinedum [9] introduced a mathematical model to represent mode coupling in UPM machines. He showed that residual vibrations and transmissibility can be reduced by selecting the proper isolator, motor, and work surface heights. He further demonstrated that coupling vibrations mode provides optimal conditions for vibrations reduction [10]. However, the vibration modes of multi degree of freedom system can also be coupled in nonlinear system due to the nonlinear stiffness. Weakly nonlinear systems can be solved by approximating the solution using perturbation methods [11], [12].

The linear vibrations of UPM machines with mode coupling due to misalignment between isolators and C.G have been reported in previous literatures. However, no work has been reported on the nonlinear coupled system of UPM machines. This is the focus of the present paper. The coupled nonlinear equations of motion of the isolated machine are presented and the solutions are obtained using the multiple scale method. The effect of changing the misalignment on the vibration amplitude and nonlinear frequency is also studied.

MATHEMATICAL MODEL

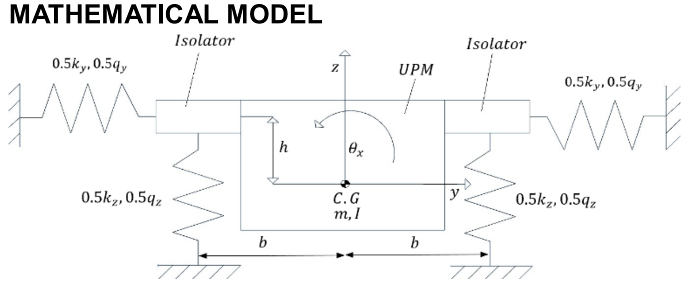

Fig. 1. Simple model of isolated machine with quadratic nonlinearity.

Fig. 1 illustrates a schematic diagram of an isolated UPM machine modeled in *y-z* plane. This machine has a mass *m* and a centroidal moment of inertia *I*. The isolators are assumed to have both linear and quadratic nonlinear stiffness. k_y, k_z and q_y, q_z are the combined linear stiffness and the nonlinear stiffness in the *y* and *z* directions, respectively. The isolator mounting is located vertically at *h* and horizontally at *b* from the center of gravity. The two coordinate *y* and θ_x are coupled through *h*. The dynamic in the *z*-direction is ignored, since the goal of this study is studying the effect *h* on the coupling in the isolated system. Moreover, the more sensitive axis in UPM is the horizontal axis.

In order to obtain the natural frequencies of the coupled system and study the effect of changing *h* on them, the damping and inertia forces are neglected. Based on that, the equation of motion for this free undamped system is

$$M\ddot{u} + Ku + Qu_1 = 0 \quad (1)$$

where *M, K, Q, u, u_1* are the mass matrix, the linear stiffness matrix, the nonlinear stiffness vector, the linear displacement vector, and the nonlinear displacement vector, respectively. these elements are defined as:

$$M = \begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix}; K = \begin{bmatrix} k_y & -k_y h \\ -k_y h & k_\theta + k_y h^2 \end{bmatrix}; \quad (2)$$

$$Q = \begin{bmatrix} q_y & -q_y h^2 \\ -q_y h & q_\theta + q_y h^3 \end{bmatrix}; u = \begin{bmatrix} y \\ \theta_x \end{bmatrix}; \; u_1 = \begin{bmatrix} y^2 \\ \theta_x^2 \end{bmatrix}$$

where

$$k_\theta = b^2 k_z; \; q_\theta = b^3 q_z \quad (3)$$

A. Perturbation Analysis

An approximation to the solution of Eq (1) can be obtained by using perturbation methods (method of multiple scales). we assume the expansion for the displacement in the form

$$y = \epsilon y_1(T_0, T_1) + \epsilon^2 y_2(T_0, T_1) + \cdots \quad (4)$$

$$\theta_x = \epsilon \theta_1(T_0, T_1) + \epsilon^2 \theta_2(T_0, T_1) + \cdots \quad (5)$$

where ϵ is a bookkeeping parameter. T_0 and T_1 are fast time scale, and slow time scale, respectively. They are defined as

$$T_0 = t \; ; \; T_1 = \epsilon t \quad (6)$$

The second derivative of time in terms of T_n is

$$(\cdot\cdot) = D_0^2 + 2\epsilon D_0 D_1 \quad (7)$$

where $D_n = \frac{\partial}{\partial T_n}$.

By substituting Eqs. (4) - (7) into Eq (1) and equating the coefficient of like powers yields

Order ϵ:

$$m D_0^2 y_1 + k_y y_1 - k_y h \theta_1 = 0 \quad (8)$$

$$I D_0^2 \theta_1 - k_y h y_1 + (k_\theta + k_y h^2)\theta_1 = 0 \quad (9)$$

Order ϵ^2:

$$m D_0^2 y_2 + k_y y_2 - k_y h \theta_2 = -2 D_0 D_1 m y_1 - q_y y_1^2 + q_y h^2 \theta_1^2 \quad (10)$$

$$I D_0^2 \theta_2 - k_y h y_2 + (k_\theta + k_y h^2)\theta_2 = -2 D_0 D_1 I \theta_1 + q_y h y_1^2 - (q_\theta + q_y h^3)\theta_1^2 \quad (11)$$

Since the problem at order ϵ is linear, the solution of Eqs (8) – (9) can be given as

$$y_1(T_0, T_1) = \Lambda_1 A_1(T_1) e^{j\omega_1 T_0} + \Lambda_2 A_2(T_1) e^{j\omega_2 T_0} + cc \quad (12)$$

$$\theta_1(T_0, T_1) = A_1(T_1) e^{j\omega_1 T_0} + A_2(T_1) e^{j\omega_2 T_0} + cc \quad (13)$$

where cc represents the complex conjugate of the preceding terms. Substituting Eqs (12) – (13) into Eqs (8) – (9), yields

$$\omega_1 = \sqrt{\frac{A - B}{2mI}}; \; \omega_2 = \sqrt{\frac{A + B}{2mI}} \quad (14)$$

$$\Lambda_i = h + \frac{k_\theta - \omega_i^2 I}{k_y h} \quad (15)$$

where

$$A = m k_\theta + m k_y h^2 + k_y I; \quad (16)$$

$$B = \sqrt{A^2 - 4mI k_y k_\theta}$$

Substituting Eqs (12) – (13) into order ϵ^2 nonlinear equations (Eqs (10) – (11)) gives

$$\begin{aligned} m D_0^2 y_2 + k_y y_2 - k_y h \theta_2 &= -2jm\Lambda_1 A_1' \omega_1 e^{j\omega_1 T_0} - 2jm\Lambda_2 A_2' \omega_2 e^{j\omega_2 T_0} \\ &+ (h^2 - \Lambda_1^2) A_1^2 q_y e^{2j\omega_1 T_0} + (h^2 - \Lambda_2^2) A_2^2 q_y e^{2j\omega_2 T_0} \\ &+ 2(h^2 - \Lambda_1\Lambda_2) A_1 A_2 q_y e^{j(\omega_1+\omega_2)T_0} + 2(h^2 - \Lambda_1\Lambda_2)\bar{A}_1 A_2 q_y e^{j(\omega_2-\omega_1)T_0} \\ &- 2q_y A_1 \bar{A}_1 \Lambda_1^2 - 2q_y A_2 \bar{A}_2 \Lambda_2^2 + 2q_y h^2 A_1 \bar{A}_1 + 2q_y h^2 A_2 \bar{A}_2 \end{aligned} \quad (17)$$

$$\begin{aligned} I D_0^2 \theta_2 - k_y h y_2 + (k_\theta + k_y h^2)\theta_2 &= -2jI A_1' \omega_1 e^{j\omega_1 T_0} - 2jI A_2' \omega_2 e^{j\omega_2 T_0} \\ &+ (q_y h \Lambda_1^2 - q_\theta - q_y h^3) A_1^2 e^{2j\omega_1 T_0} + (q_y h \Lambda_2^2 - q_\theta - q_y h^3) A_2^2 e^{2j\omega_2 T_0} \\ &+ 2(q_y h \Lambda_1 \Lambda_2 - q_\theta - q_y h^3) A_1 A_2 e^{j(\omega_1+\omega_2)T_0} \\ &+ 2(q_y h \Lambda_1 \Lambda_2 - q_\theta - q_y h^3) \bar{A}_1 A_2 e^{j(\omega_2-\omega_1)T_0} \\ &+ 2(q_y h \Lambda_1^2 - q_\theta - q_y h^3) A_1 \bar{A}_1 + 2(q_y h \Lambda_2^2 - q_\theta - q_y h^3) A_2 \bar{A}_2 \end{aligned} \quad (18)$$

A. Non-Resonant Case

In this case the natural frequencies are not commensurable. Based on that, the solvability conditions in Eqs (17) – (18) yield

$$A_n' = 0 \xrightarrow{yields} A_n = a_n e^{j\beta_n} \quad (19)$$

where a_n and β_n are real constants and the nonlinear frequency equals the linear one.

B. The Resonant Case (Internal Resonance)

In this case we assume that the natural frequencies are commensurable or near commensurable *($\omega_2 \approx 2\omega_1$)*. The detuning parameter σ can be introduced according to

$$\omega_2 = 2\omega_1 + \epsilon\sigma \quad (20)$$

In order to introduce the solvability conditions for Eqs (17) - (18), the solution of y_2 and θ_2 should be introduced as [11], [12]

$$y_2 = P_{11} e^{j\omega_1 T_0} + P_{12} e^{j\omega_2 T_0} \quad (21)$$

$$\theta_2 = P_{21} e^{j\omega_1 T_0} + P_{22} e^{j\omega_2 T_0} \quad (22)$$

Substituting Eqs (21) - (22) into Eqs (17) – (18), equating the coefficient of $e^{j\omega_1 T_0}$ and $e^{j\omega_2 T_0}$ on both sides, and equating the determinant of the coefficient matrix to zero, the solvability conditions can be obtained as

$$R_{2n} = -\Lambda_n R_{1n} \quad (23)$$

where

$$R_{11} = -2jm\Lambda_1 A_1' \omega_1 + 2(h^2 - \Lambda_1\Lambda_2)\bar{A}_1 A_2 q_y e^{j\sigma T_1} \quad (24)$$

$$R_{21} = -2jI\Lambda_1 A_1' \omega_1 + 2(q_y h \Lambda_1 \Lambda_2 - q_\theta - q_y h^3)\bar{A}_1 A_2 e^{j\sigma T_1} \quad (25)$$

$$R_{12} = -2jm A_2' \omega_2 + (h^2 - \Lambda_1^2) A_1^2 q_y e^{-j\sigma T_1} \quad (26)$$

$$R_{22} = -2jI A_2' \omega_2 + 2(q_y h \Lambda_1 \Lambda_2 - q_\theta - q_y h^3)\bar{A}_1 A_2 e^{-j\sigma T_1} \quad (27)$$

Introducing $A_n = a_n e^{j\beta_n}$ with real a_n and β_n in Eqs (23) - (27), and separating the real and imaginary parts lead to

$$\beta_1' = -\frac{a_2}{2}\Gamma_1 \cos\gamma \tag{28}$$

$$a_1' = \frac{a_1 a_1}{2}\Gamma_1 sin\gamma \tag{29}$$

$$a_2\beta_2' = -\frac{a_1^2}{4}\Gamma_2 \cos\gamma \tag{30}$$

$$a_2' = -\frac{a_1^2}{4}\Gamma_2 sin\gamma \tag{31}$$

where

$$\gamma = \beta_2 - 2\beta_1 + \sigma T_1 \tag{32}$$

$$\Gamma_1 = \frac{q_y(h^2\Lambda_1 + h\Lambda_1\Lambda_2 - \Lambda_1^2\Lambda_2 - h^3) - q_\theta}{I\omega_1 + m\Lambda_1^2\omega_1} \tag{33}$$

$$\Gamma_2 = \frac{q_y(h^2\Lambda_2 + h\Lambda_1^2 - \Lambda_1^2\Lambda_2 - h^3) - q_\theta}{I\omega_2 + m\Lambda_2^2\omega_2} \tag{34}$$

Dividing Eq(29) by Eq(31) yields to

$$a_1' a_1 + \upsilon a_2' a_2 = 0 \tag{35}$$

where

$$\upsilon = \frac{2\Gamma_1}{\Gamma_2} \tag{36}$$

Integrating Eq(35) leads to

$$a_1^2 + \upsilon a_2^2 = E \tag{37}$$

where E is the constant of integration. The results is Eq(36) indicate that the solution is bounded when v is positive, or in other words when both $\Gamma_1\Gamma_2$ have the same sign. In this study, we are going to study only the bounded case. Deriving Eq(32) with respect to T_1, changing the independent variable T_1 to a_2 by using Eqs(28),(30)&(31), and integrating the result, we obtain

$$a_1^2 a_2 \cos\gamma - 2\sigma a_2^2 = L \tag{38}$$

where L is the constant of integration. If we let

$$a_1^2 = E\xi \tag{39}$$

a_2 becomes

$$a_2^2 = \frac{E(1-\xi)}{\upsilon} \tag{40}$$

By substituting Eqs (38) - (40) into Eq (29), the following equation is obtained

$$\frac{\upsilon}{E}\left(\frac{d\xi}{dT_1}\right)^2 = F^2(\xi) - G^2(\xi) \tag{41}$$

where

$$F(\xi) = \pm\xi\sqrt{1-\xi} \tag{42}$$

$$G(\xi) = \pm\sqrt{\frac{\upsilon}{E^3}}[L + 2\sigma E(1-\xi)] \tag{43}$$

In this study, the curve G is assumed to meet F at three distinct points, such that $\xi_1 \le \xi_2 \le \xi_3$. In this case ξ is periodic, but the motion is not. Based on that, the solution of ξ can be defined in terms of Jacobi elliptic functions. Eq(41) can be rewritten as

$$\frac{\upsilon}{E}\left(\frac{d\xi}{dT_1}\right)^2 = (\xi_3 - \xi)(\xi - \xi_2)(\xi - \xi_1) \tag{44}$$

By introducing the following transformation,

$$\xi_3 - \xi = (\xi_3 - \xi_2)sin^2\chi \tag{45}$$

and integrating Eq(44) after applying this transformation, we obtain

$$\xi = \xi_3 - (\xi_3 - \xi_2)sn^2[\kappa(t - t_0);\eta] \tag{46}$$

where sn is Jacobi elliptic function, t_0 refers to χ=0, and κ,η are

$$\kappa = \epsilon\sqrt{\frac{E(\xi_3 - \xi_1)}{\upsilon}} \tag{47}$$

$$\eta = \sqrt{\frac{\xi_3 - \xi_2}{\xi_3 - \xi_1}} \tag{48}$$

Using Eqs (39) - (40) a_1 and a_2 become

$$a_1 = \sqrt{E(\xi_3 - (\xi_3 - \xi_2)sn^2[\kappa(t - t_0);\eta])} \tag{49}$$

$$a_2 = \sqrt{\frac{E}{\upsilon}(1 - (\xi_3 - (\xi_3 - \xi_2)sn^2[\kappa(t - t_0);\eta]))} \tag{50}$$

The nonlinear frequency *(ω_{nl}=ω+β)* for this system are obtained as

$$\omega_{nl1} = \omega_1 - \frac{a_2}{2}\Gamma_1 \tag{51}$$

$$\omega_{nl2} = \omega_2 - \frac{a_1^2}{4a_2}\Gamma_2 \tag{52}$$

It is obvious from Eqs(49) - (52) that the nonlinear frequencies are not constant and vary with time, therefore, the motion is aperiodic when the modes are coupled due to internal resonance.

II. NUMERICAL RESULTS

The numerical simulation is carried out using the values from [8] listed in Table I.

TABLE I

Key parameters of UPM machine

Parameter	***value***
m (kg)	*1182*
I_x (kg.m^2)	*96*
k_y (KN/m)	*880*
k_z (KN/m)	*1,200*
q_y (KN/m^2)	*100*
q_z (KN/m^2)	*400*
b_y (mm)	*295*

Fig. 2 shows the effect of changing the isolators position on *v*. it is observed that *v* has positive values for *h= -0.2 – 0 m*, and negative values for *h= 0 – 0.2 m*. since our solution is only valid for the positive values of *v*, our study is focused on the negative range of *h*. The peak at zero represents discontinuous point because there is no coupling between the modes.

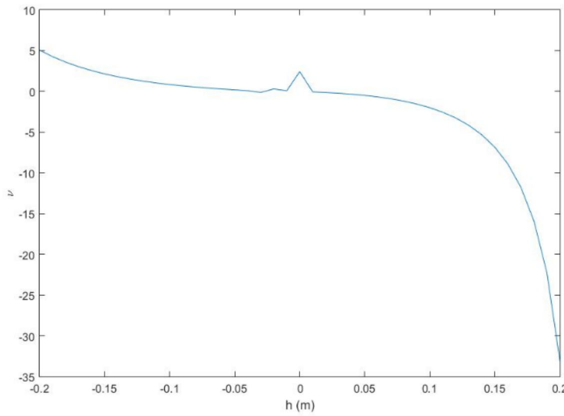

Fig. 2 The effect of changing h on v.

Fig. 3 depicts the relation between *h* and the value of detuning parameter. The results show that the detuning parameter increases with increasing absolute value of *h*, and the zero mistuning occurs when *h=-0.213 m.*

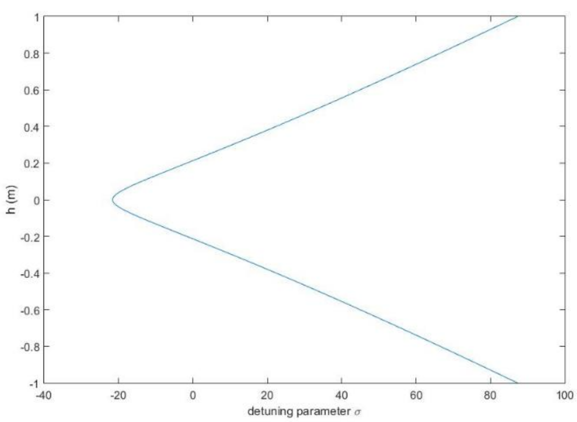

Fig. 3 The effect of h on detuning parameter σ.

Figs. 4&5 illustrate the modal amplitudes a_1 and a_2 with time for two different initial conditions. The results indicate that the solution of quadratic nonlinear equation depends on the initial conditions. It is observed that the amplitude of the second mode has been significantly affected by the nonlinearity with time, when the initial condition of this mode is zero. Furthermore, the modal amplitudes depend on the value of *h,* too, as shown in Figs. 6&7. The results in Fig.6 indicate that the mode amplitude a_1 increases with increasing absolute value of *h*. The opposite is observed in Fig.7 for the second mode amplitude a_2.

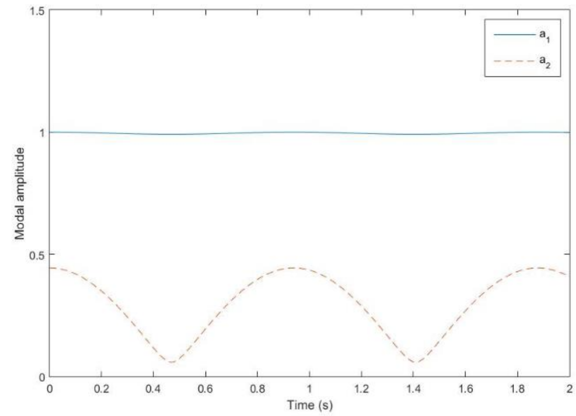

Fig. 4. Free oscillation amplitudes $a_1(0)$ =1, $a_2(0)$ =0.

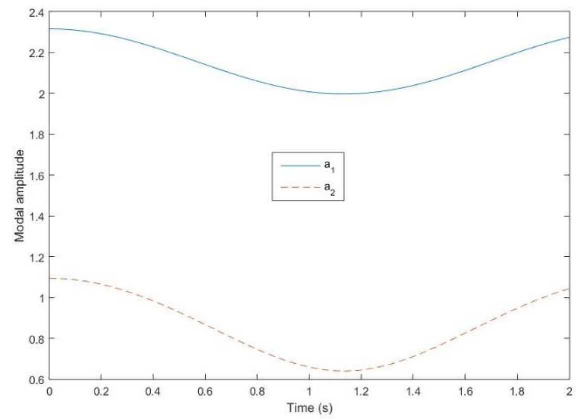

Fig. 5 Free oscillation amplitudes $a_1(0)$ =1, $a_2(0)$ =1.

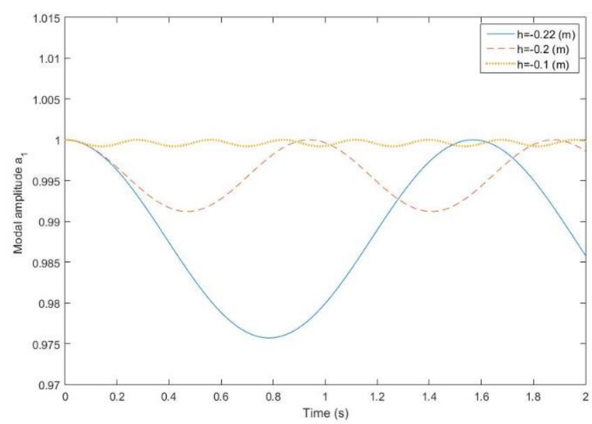

Fig. 6 The effect of changing h on a_1, $a_1(0)$ =1, $a_2(0)$ =0.

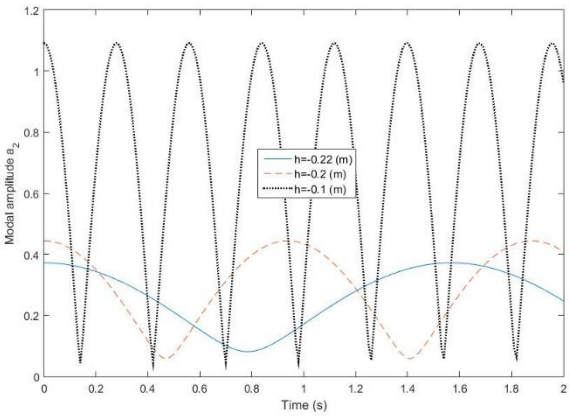

Fig. 7 The effect of changing h on a_2, $a_1(0)$ =1, $a_2(0)$ =0.

Figs. 8&9 depict the changing of nonlinear frequencies with time. The results show that the behavior of nonlinear frequency depends on the value of initials condition of each mode. Moreover, it is observed that the modal amplitude fluctuates more when the initial condition of second mode is zero, comparing to the linear frequencies. Furthermore, the nonlinear frequency depends also on the value of *h* as shown in Fig.10. It is also observed that the nonlinear frequencies of two modes come closest to each other as the misaligning decreases. This result corroborate those obtained for the linear frequencies in [7], [9].

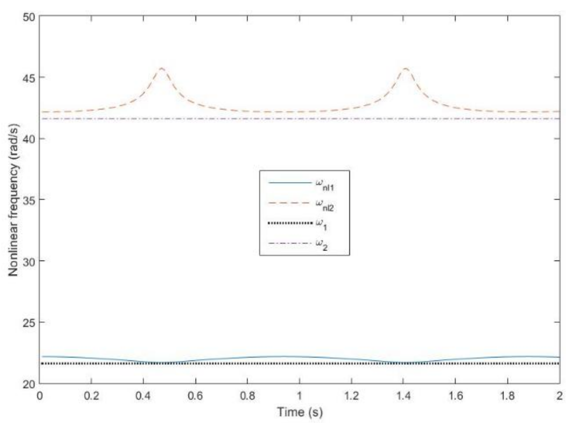

Fig. 8. The effect of nonlinearity on frequency, $a_1(0)$ =1, $a_2(0)$ =0.

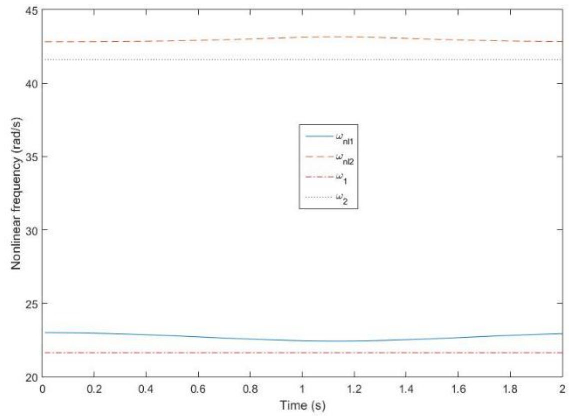

Fig. 9 The effect of nonlinearity on frequency, $a_1(0)$ =1, $a_2(0)$ =1.

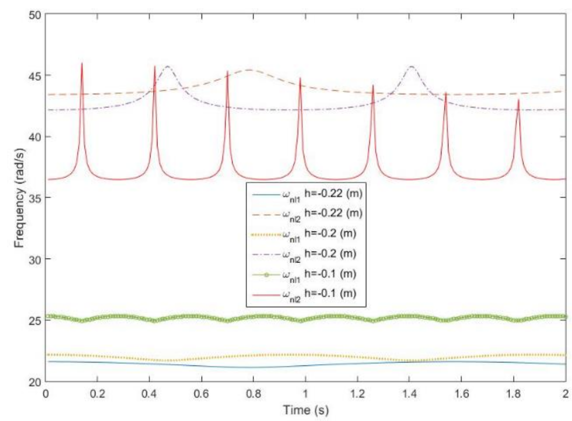

Fig. 10 The effect of h on nonlinear frequency, $a_1(0)$ =1, $a_2(0)$ =0.

CONCLUSION

In this paper, the nonlinear vibration of free undamped ultra-precision manufacturing machine (UPM) has been investigated analytically. The passive isolator is assumed to have a combination between linear and quadratic nonlinear stiffness. The modes are coupled linearly due to altering the location of isolators with respect to the center of gravity of the UPM. They are also coupled linearly due to the nonlinear resonance ($\omega_2 \approx 2\omega_1$). The closed form solution for the bounded case ($v > 0$) is obtained by using the method of multiple scales, and the final solution is presented in terms of Jacobi elliptic functions. The non-resonant case shows that there is no nonlinear coupling, and the amplitudes are constant. On the other hand, the internal resonance case shows a nonlinear coupling between the amplitudes. The results of the numerical analysis demonstrate that both the amplitudes and frequencies depend on the initial conditions and the distance between isolator and center of gravity. Moreover, the results indicate that increasing the isolator support height *h* increases the value of a_2 and reduces a_1.

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