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ABSTRACT

Prior research has investigated resonators capable of self-tuning through the use of a sliding mass. This passive tuning mechanism can be utilized to improve vibration control; however, little is known about the nonlinear dynamic interactions between the vibrating beam and sliding mass, particularly as these apply to vibration energy harvesting applications. This paper investigates this problem by numerically and experimentally examining the response of an electromagnetic self-tuning energy harvester. We present the governing equations of this electromagnetic cantilever beam with a sliding mass using the extended Hamilton principle. These equations are then discretized using the Galerkin method and solved numerically. An experiment is carried out to validate the numerical analysis. Parametric studies are conducted to examine the effect of different system parameters on the performance of the self-tuning harvester.

In modern society, there has been a significant increase in energy demands, making energy a crucial global issue. However, an extensive use of conventional power resources is a cause of other global issues, namely, environmental concerns. Furthermore, these sources are finite and will eventually be exhausted. On the other hand, there are numerous obstacles to replacing the chemical batteries that are currently employed in many engineering applications. Hence, the study of robust systems that can harvest clean and renewable energy is an area of significant interest to researchers.[1]

Vibration energy harvesters (VEHs) provide useful power generation in many circumstances due to the presence of vibrations from many sources including civil structures, human movement, vehicles, power lines, and numerous others. In general, vibrations can be harvested by piezoelectric materials or electromagnetic transducers.[2] VEHs are effective when the harvester's resonant frequency is tuned or close to the source (excitation) frequencies. However, when this is not the case, effectiveness rapidly drops for even a small change in excitation frequency.[2–12] Therefore, there is a crucial need for systems that can resonate at a wider range of frequencies since environmental vibration sources have varied frequencies that also vary over time. This can be achieved by several methods, including active or passive frequency tuning. The former is effective but requires input power, whether through sacrificing a significant portion of the harvested power or even using an external power source. On the other hand, passive systems can tune themselves without electrical power. One passive solution is via a sliding mass attachment that can move along the beam and settle in a position such that the resonance frequency of the system equals the excitation frequency.[6–24] These self-tuning systems have demonstrated a significant enhancement to energy harvesting, allowing harvesting over a wider frequency band due to their ability to passively tune the system's resonant frequency.

Theoretical studies for self-resonant (self-tuning) systems of various structural types can be found in the literature. For instance, the string model was studied by Boudaoud *et al.*,[22] flexural vibration of beams was studied by Miller and co-workers,[15,16,20] and plates were studied by Wang and Lo.[23] In addition, experimental demonstrations of self-resonant systems have been detailed in numerous papers.[6,7,10,16–18,20] For beam structures, systems with different boundary conditions have also been investigated: for instance, a clamped–clamped system was investigated by Miller and co-workers,[10,13,16–18] while a free-end system was studied by Mori and co-workers.[6,7,20,21] Moreover, Krack *et al.* developed further understanding of the dynamics of a self-resonant system through interpreting experimental results based on nonlinear dynamics.[14] Furthermore, an array of cantilever beam models with sliding-mass systems were investigated by Staaf *et al.*[8,9,11] However, previous attempts to study self-tuning in broadband energy harvesting systems were largely concentrated on piezoelectric harvesters, which are limited to small scale applications. To the best of our knowledge, there is no work in the literature that investigates the dynamic coupling of a self-tuning system with an electromagnetic harvester. This is the focus of the present study.

In this paper, we investigate the dynamics of a self-tuning cantilever beam with an electromagnetic harvester: in particular, the effect of the sliding mass on the output harvested power. We obtain the nonlinear governing equations of motion for the system and the harvester in the form of three coupled differential equations and discretize these partial differential equations for numerical simulation by Galerkin projection. The resulting coupled system of ordinary differential equations is integrated numerically, solving for the response of the beam and slider and calculating the harvested power. We also investigate the system experimentally to validate these analytical results and observations.

A schematic for the proposed self-tuning VEH is shown in Fig. 1. This VEH consists of a cantilever beam with a sliding mass and electromagnetic VEH system attached to the tip mass. The system's cantilever beam has mass per unit length m, bending stiffness EI, and length L. The transverse displacement of the beam is defined by $\bar{w}(x,t)$. A slider is installed on the beam with mass M. This slider slides along the beam, and its position $\bar{s}$ is measured from the fixed end of the beam. On the free end of the beam, a tip mass M_t is attached to the beam. This mass has a ferromagnetic core with coils wrapped around the core to construct the electromagnetic VEH. The coils have internal inductance L_i. The tip mass of the VEH vibrates into an electromagnetic field generated by two magnets with electromagnetic coupling coefficient k_f. To harvest power, the VEH system is shunted to a circuit with resistance R. When the proposed VEH is subjected to a base excitation $\ddot{w}_0$, the governing equations of motion of this self-adaptive VEH system can be obtained as follows:

$$m\ddot{\bar{w}} + EI\bar{w}'''' + M[\ddot{\bar{w}} + 2\dot{\bar{s}}\dot{\bar{w}}' + \ddot{\bar{s}}\bar{w}' + \dot{\bar{s}}^2\bar{w}'']_{x=\bar{s}} = -m\ddot{w}_0 - M\ddot{w}_0\delta(x-\bar{s}) - M_t\ddot{w}_0\delta(x-L) + k_f\dot{\bar{q}}\delta(x-L), \quad (1)$$

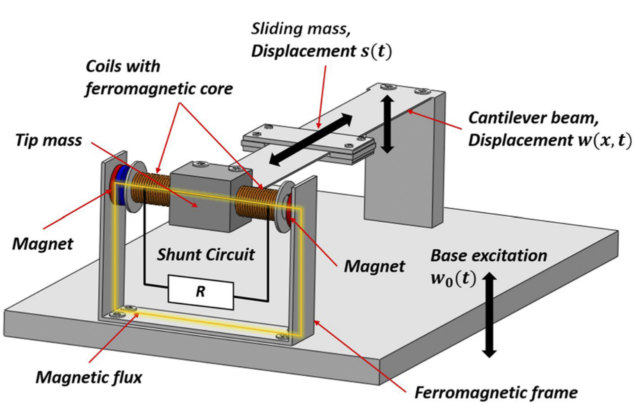

FIG. 1. Key components of the self-tuning vibration energy harvesting system.

$$L_i\ddot{\bar{q}} - k_v\dot{\bar{w}} + R\dot{\bar{q}} = 0, \quad (2)$$

$$M\ddot{\bar{s}} + M\bar{w}'[\ddot{\bar{w}} + 2\dot{\bar{s}}\dot{\bar{w}}' + \ddot{\bar{s}}\bar{w}' + \dot{\bar{s}}^2\bar{w}'']_{x=\bar{s}} = -M\bar{w}'\ddot{w}_0\delta(x-\bar{s}), \quad (3)$$

where $\bar{q}$ is the charge, δ is the Dirac delta function, and dots $(\cdot)$ and primes $(')$ represent the partial derivatives with respect to time and position, respectively.

Equations (1)–(3) can be rewritten in dimensionless form as

$$\ddot{w} + \omega_n^2 w'''' + m_1[\ddot{w} + 2\dot{s}\dot{w}' + \ddot{s}w' + \dot{s}^2 w'']_{\zeta=s} = -\ddot{w}_0 - m_1\ddot{w}_0\delta(\zeta-s) - m_2\ddot{w}_0\delta(\zeta-1) + \dot{q}\delta(\zeta-1), \quad (4)$$

$$\ddot{q} - \alpha_1\dot{w} + \alpha_2\dot{q} = 0, \quad (5)$$

$$\ddot{s} + w'[\ddot{w} + 2\dot{s}\dot{w}' + \ddot{s}w' + \dot{s}^2 w'']_{\zeta=s} = -w'\ddot{w}_0\delta(\zeta-s), \quad (6)$$

where $\omega_n^2 = \frac{EI}{mL^4}$, $m_1 = \frac{M}{m}$, $m_2 = \frac{M_t}{m}$, $w = \bar{w}/L$, $s = \bar{s}/L$, $\zeta = \bar{x}/L$, $q = \bar{q}/\alpha$, $\alpha = \frac{m\omega_n L}{k_f}$, $\alpha_1 = \frac{k_v L}{L_i\omega_n\alpha}$, and $\alpha_2 = \frac{R}{L_i\omega_n}$.

Since L_i is typically small, one can get the following from Eq. (6):

$$\dot{q} = \beta\dot{w}, \quad (7)$$

where $\beta = \alpha_1/\alpha_2$. Therefore, Eq. (4) can be rewritten as

$$\ddot{w} + \omega_n^2 w'''' + m_1[\ddot{w} + 2\dot{s}\dot{w}' + \ddot{s}w' + \dot{s}^2 w'']_{\zeta=s} = -\ddot{w}_0 - m_1\ddot{w}_0\delta(\zeta-s) + m_2\ddot{w}_0\delta(\zeta-1) + \beta\dot{w}\delta(\zeta-1). \quad (8)$$

Upon applying the Galerkin projection [i.e., $w = \sum_{n=1}^{\infty} = W_n(\zeta)T_n(\tau)$, where $W_n(\zeta)$ is the normalized eigenfunction and $T_n(\tau)$ is an arbitrary function of time], Eqs. (6) and (8) can be discretized by using the instantaneous mode shape of the beam and the slider. Upon considering the first three modes in the expansion, the resulting system of ordinary differential equations can then be solved numerically using MATLAB's built-in integrator ODE45. For a harmonic input force, the Galerkin discretization with three modes shows good convergence when compared to the five-mode expansion case, with an error of only $\sim 10^{-6}$. This is because we are considering frequencies within the lowest two modes in the present study. In this work, we focus our analyses near the fundamental frequency of the system to demonstrate the increase in the frequency band for the VEH. However, a similar increase in the frequency band is predicted at excitation frequencies close to higher frequencies of the system. A higher number of modes will need to be considered in order to study higher excitation frequencies. In this paper, we simulate a prototype with parameters defined in Table I.

TABLE I. VEH experimental system parameters.

Parameter	Value
Beam length (L)	148 mm
Beam bending stiffness (EI)	0.1109 N m^2
Mass per unit length of the beam (m)	0.0721 Kg/m
Mass of the tip mass (M_t)	34.15 g
Mass of the sliding mass (M)	14.23 g
Electromagnetic coupling coefficient	0.9092 V/N
Shunted resistor	0.2 Ω

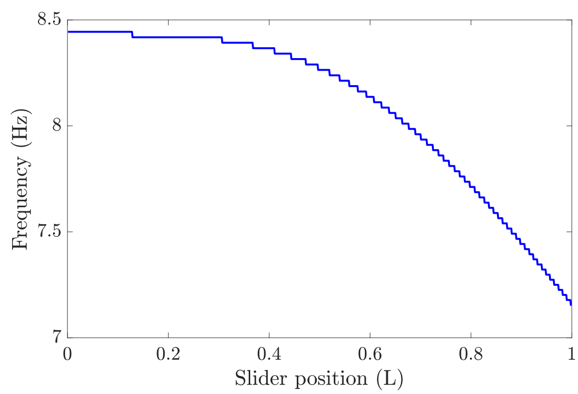

FIG. 2. Fundamental natural frequency of the system varies between 7.2 Hz and 8.4 Hz with changing slider position.

As the slider moves along the beam, the frequency of the system changes based on the instantaneous position of the slider, as shown in Fig. 2. The results demonstrate that the system's fundamental frequency varies between 7.2 Hz and 8.4 Hz as the slider moves along the beam due to external disturbances. This frequency range can be further adjusted by the weight of the sliding mass. When the system is subjected to an external force with a frequency in this range, the slider moves along the beam and meets the resonance frequency of the system along its way toward the vibration anti-node. This results in a significant increase in the harvested power and, thus, increases the operational frequency range of our VEH as compared to conventional energy harvesters.

Based on the initial position of the slider and the excitation frequency, the slider moves to settle down on a vibration node or an anti-vibration node. Slider position and power harvested over time for multiple excitation frequencies are depicted in Fig. 3. For the moving slider case, the results indicate that power in the order of mW can be harvested through the self-adaptive harvester, as shown in Figs. 3(a) and 3(c). For excitation frequencies inside the frequency range in Fig. 2, the slider moves from the initial position in the middle of the beam toward the free end of the beam and the harvested power increases. However, at frequencies above this range, the slider moves toward the fixed end, as shown in Fig. 3(b). The self-adaptive VEH can harvest a significant amount of power as the slider moves along the beam. This harvested power is much higher than that of a fixed slider VEH ($s_0 = 0.5L$) at these frequencies, as shown in Figs. 3(a) and 3(c). Although we focus our analyses around the fundamental frequency, the system can track the vibration node or anti-node at higher resonance frequencies based on the excitation

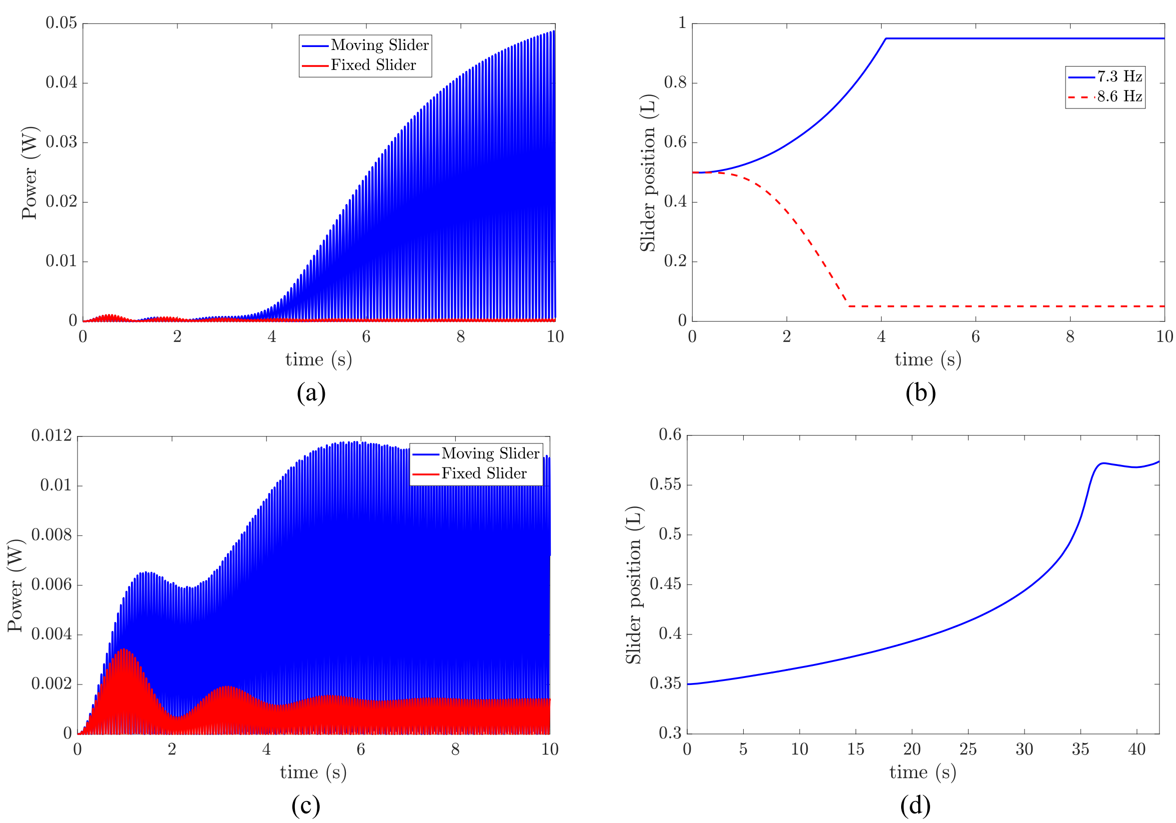

FIG. 3. Simulation results at different excitation frequencies: (a) Harvested power at 7.3 Hz; (b) slider position at 7.3 Hz and 8.6 Hz; (c) harvested power at 8.6 Hz; and (d) slider position at frequencies close to the second vibration mode at 72.2 Hz.

frequency. Figure 3(d) shows that the slider moves toward the vibration anti-node at excitation frequencies close to the second natural frequency, eventually settling at the anti-node.

To demonstrate the improvement in power generation by the self-adaptive VEH, we excite the moving and fixed slider versions at different frequencies with the initial slider position in the middle of the beam, then integrate the harvested power over the transient time of 10 s, and plot the results in Fig. 4. This figure shows that the self-adaptive VEH can harvest more power over a wider range of frequencies. Therefore, the self-adaptive VEH shows a superior performance at a wider frequency band near the fundamental frequency of the system. This superiority in performance also stands at higher frequencies.

Experiments are carried out to validate the analytical model and demonstrate the effect of a slider on the self-tuning of a VEH system. In these experiments, the response of the beam and sliding mass are studied over a period of time for moving and fixed slider VEHs at different excitation frequencies.

The experimental setup is shown in Fig. 5. We collected data using an aluminum cantilever (clamped-free) beam system with a tip mass and slider depicted in Fig. 1. The tip mass is attached to the beam using screws, and two wire coils with ferromagnetic cores are attached to the sides of the tip mass.

The harvester was mounted on a Vibration Test Systems' dynamic shaker (VG-100) such that the beam's vibration is horizontal. A laser displacement sensor (Micro-Epsilon Opto NCDT) pointed at the beam's free end from one side recorded the horizontal amplitude of the beam's tip vibration. Additionally, a laser vibrometer (Polytec PSV-500) aimed along the length of the beam measured the position of the mass sliding along the beam when subjected to vibrations. The base excitation was measured using an accelerometer (PCB 356A16), and the harvested power was measured by shunting to an external resistor. Specific baseline parameters for the VEH experimental system are similar to those used in numerical simulations.

For frequencies below the range defined in Fig. 2, the slider moves toward the antinode, as depicted in Fig. 6. This shows a good agreement in the motion trend with the analytical results shown in Fig. 3(b). For higher frequencies, the slider does not move at small excitation amplitude since the nonlinear coupling term [i.e., the second term in Eq. (3)] cannot overcome the static friction, as shown in Fig. 7. However, the nonlinear coupling term overcomes the static friction at higher excitation amplitudes. This, in turn, forces the slider to move toward the fixed end, as shown in Fig. 7. Therefore, the experimental results show a good agreement with the analytical results shown in Fig. 3(b) at higher excitation frequencies. The experimental measurements show that the slider takes more time to travel to the free end compared to the analytical results. This is because friction and backlash are not considered in the analytical model.

In addition, the analytical model did not include the damping between the slider and host beam, as identifying this damping was beyond the scope of the current paper. Thus, an exact quantitative match was not obtained between the analytical and experimental results. However, a good qualitative match between the analytical model and experimental results was shown as described above. This qualitative match supports our conclusion about the

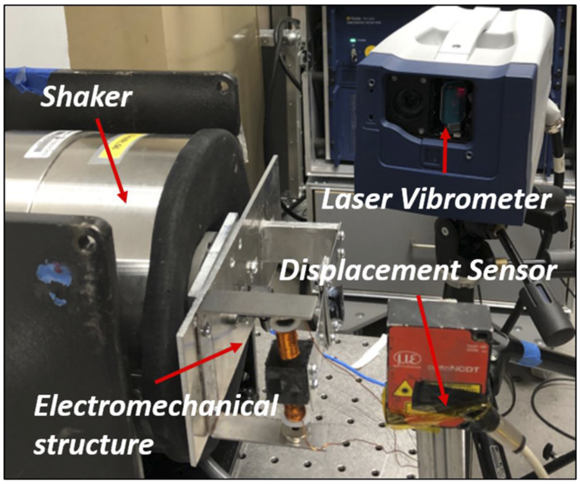

FIG. 5. Key components of the experimental setup.

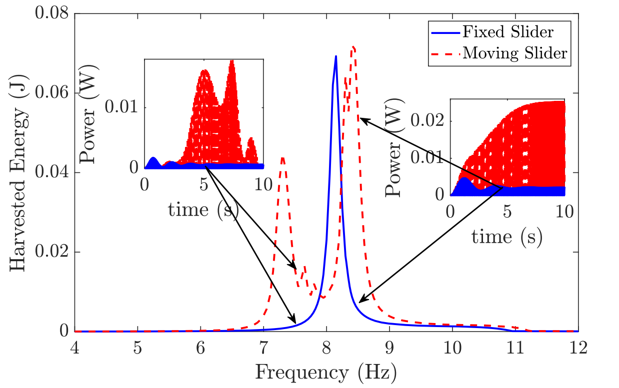

FIG. 4. Total energy harvested over 10 s with and without a sliding mass for a range of frequencies $s_0 = 0.5L$.

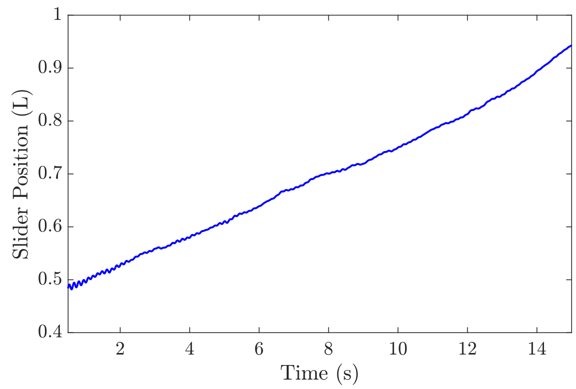

FIG. 6. Measured position of the slider at a low excitation frequency (8.25 Hz).

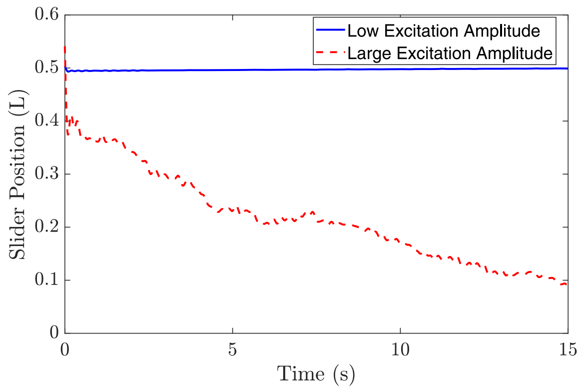

FIG. 7. Measured position of the slider at a high excitation frequency for low and high excitation amplitudes (10.25 Hz).

self-tuning mechanism, showing a dramatic enhancement in the frequency bandwidth and energy harvesting.

In conclusion, this article details the analysis and modeling of a self-tuning electromagnetic vibration energy harvester. This harvester takes the form of a cantilever beam with electromagnetic tip mass, with self-tuning facilitated by the presence of a sliding mass. This mass is free to move along the beam, changing the system's natural frequency. We presented the nonlinear governing equations of this system and then discretized these equations in preparation for numerical solution. This solution was validated through experiments on a physical model of the system and then utilized to simulate the response of the harvester with changes in multiple parameters. The results demonstrate that a self-tuning VEH has a superior performance over conventional energy harvesters in terms of the harvested power and effective frequency range. The future work will include friction in the analytical formulation, and a thorough nonlinear analysis will be conducted to understand the nature of limit cycles and bifurcation of the self-tuning harvester.

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DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding authors upon reasonable request.

REFERENCES

[1] A.-N. Moh'd A, M. Bukhari, and M. Mansour, "A combined CPV/T and ORC solar power generation system integrated with geothermal cooling and electrolyser/fuel cell storage unit," Energy **133**, 513–524 (2017).
[2] S. Priya and D. J. Inman, *Energy Harvesting Technologies* (Springer, 2009), Vol. 21.
[3] L. Zuo and X. Tang, "Large-scale vibration energy harvesting," J. Intell. Mater. Syst. Struct. **24**, 1405–1430 (2013).
[4] X. Tang and L. Zuo, "Simultaneous energy harvesting and vibration control of structures with tuned mass dampers," J. Intell. Mater. Syst. Struct. **23**, 2117–2127 (2012).
[5] I. L. Cassidy, J. T. Scruggs, and S. Behrens, "Design of electromagnetic energy harvesters for large-scale structural vibration applications," Proc. SPIE **7977**, 79770P (2011).
[6] K. Mori, T. Horibe, S. Ishikawa, Y. Shindo, and F. Narita, "Characteristics of vibration energy harvesting using giant magnetostrictive cantilevers with resonant tuning," Smart Mater. Struct. **24**, 125032 (2015).
[7] J. Chandwani, R. Somkuwar, and R. Deshmukh, "Multi-band piezoelectric vibration energy harvester for low-frequency applications," Microsyst. Technol. **25**, 3867 (2019).
[8] L. G. H. Staaf, A. D. Smith, E. Köhler, P. Lundgren, P. D. Folkow, and P. Enoksson, "Achieving increased bandwidth for 4 degree of freedom self-tuning energy harvester," J. Sound Vib. **420**, 165–173 (2018).
[9] L. G. H. Staaf, A. D. Smith, P. Lundgren, P. D. Folkow, and P. Enoksson, "Effective piezoelectric energy harvesting with bandwidth enhancement by assymetry augmented self-tuning of conjoined cantilevers," Int. J. Mech. Sci. **150**, 1–11 (2019).
[10] P. Pillatsch, L. M. Miller, E. Halvorsen, P. K. Wright, E. M. Yeatman, and A. S. Holmes, "Self-tuning behavior of a clamped-clamped beam with sliding proof mass for broadband energy harvesting," J. Phys.: Conf. Ser. **476**, 012068 (2013).
[11] L. G. H. Staaf, E. Köhler, P. D. Folkow, and P. Enoksson, "Smart design piezoelectric energy harvester with self-tuning," J. Phys.: Conf. Ser. **922**, 012007 (2017).
[12] O. Barry and M. Bukhari, "On the modeling and analysis of an energy harvester moving vibration absorber for power lines," in *ASME 2017 Dynamic Systems and Control Conference* (American Society of Mechanical Engineers, 2017), p. V002T23A005.
[13] H. Kim, A. Smith, O. Barry, and L. Zuo, "Self-resonant energy harvester with a passively tuned sliding mass," in *Proceedings of the ASME 2019 Dynamic Systems and Control Conference* (ASME, 2019).
[14] M. Krack, N. Aboulfotoh, J. Twiefel, J. Wallaschek, L. A. Bergman, and A. F. Vakakis, "Toward understanding the self-adaptive dynamics of a harmonically forced beam with a sliding mass," Arch. Appl. Mech. **87**, 699–720 (2017).
[15] J. J. Thomsen, "Vibration suppression by using self-arranging mass: Effects of adding restoring force," J. Sound Vib. **197**, 403–425 (1996).
[16] L. M. Miller, P. Pillatsch, E. Halvorsen, P. K. Wright, E. M. Yeatman, and A. S. Holmes, "Experimental passive self-tuning behavior of a beam resonator with sliding proof mass," J. Sound Vib. **332**, 7142–7152 (2013).
[17] L. Yu, L. Tang, L. Xiong, T. Yang, and B. R. Mace, "A passive self-tuning nonlinear resonator with beam-slider structure," Proc. SPIE **10967**, 109670K (2019).
[18] L. Yu, L. Tang, and T. Yang, "Experimental investigation of a passive self-tuning resonator based on a beam-slider structure," Acta Mech. Sin. **35**, 1079–1092 (2019).
[19] N. Aboulfotoh, M. Krack, J. Twiefel, and J. Wallaschek, "A self-resonant system-experimental investigations of boundary and operating conditions," Proc. Appl. Math. Mech. **16**, 253–254 (2016).
[20] E. C. Miranda and J. J. Thomsen, "Vibration induced sliding: Theory and experiment for a beam with a spring-loaded mass," Nonlinear Dyn. **16**, 167–186 (1998).
[21] F. Khalily, M. F. Golnaraghi, and G. R. Heppler, "On the dynamic behaviour of a flexible beam carrying a moving mass," Nonlinear Dyn. **5**, 493–513 (1994).
[22] A. Boudaoud, Y. Couder, and M. Ben Amar, "A self-adaptative oscillator," Eur. Phys. J. B **9**, 159–165 (1999).
[23] Y. Wang and C. Lo, "Design of hybrid dynamic balancer and vibration absorber," Shock Vib. **2014**, 397584.
[24] M. A. Bukhari, O. Barry, and E. Tanbour, "On the vibration analysis of power lines with moving dampers," J. Vib. Control **24**, 4096–4109 (2018).